$\qquad$

## Equations with Lenses <br> SNC2D

The Lens Equation (for converging or diverging lenses) is:

$$
\frac{1}{f}=\frac{1}{d_{o}}+\frac{1}{d_{i}}
$$


where $f=$ focal length, $d_{o}=$ object distance (from the lens), and $d_{i}=$ image distance

## Example: Finding the Image Formed by a Converging Lens

A 4.0-cm tall light bulb is placed 18 cm from a converging lens having a focal length of 12 cm . Determine the image distance.

Givens: $f=12 \mathrm{~cm} \quad d_{o}=18 \mathrm{~cm} \quad$ Unknown $d_{i}=$ ?
Select an Equation: $\frac{1}{f}=\frac{1}{d_{i}}+\frac{1}{d_{o}}$ becomes $\frac{1}{d_{i}}=\frac{1}{f}-\frac{1}{d_{o}}$
Substitute and Solve: $\frac{1}{d_{i}}=\frac{1}{12 \mathrm{~cm}}-\frac{1}{18 \mathrm{~cm}}$

$$
\begin{aligned}
& \frac{1}{d_{i}}=\frac{3}{36 \mathrm{~cm}}-\frac{2}{36 \mathrm{~cm}} \quad \quad \quad \quad \text { (find a common denominator) } \\
& \frac{1}{d_{i}}=\frac{1}{36 \mathrm{~cm}} \quad \text { so } \quad d_{i}=36 \mathrm{~cm}
\end{aligned}
$$

## Practice: Finding the Image Formed by a Converging Lens

A $2.0-\mathrm{cm}$ tall candle flame is placed 6.0 cm from a converging lens having a focal length of 4.0 cm . Determine the image distance.

Givens: $\quad f=\quad d_{o}=\quad$ Unknown $d_{i}=$ ?
Select an Equation: $\frac{1}{f}=\frac{1}{d_{i}}+\frac{1}{d_{o}}$ becomes $\frac{1}{d_{i}}=\frac{1}{f}-\frac{1}{d_{o}}$
Substitute and Solve: $\frac{1}{d_{i}}=$

$$
\begin{array}{ll}
\frac{1}{d_{i}}= & \quad \text { (find a common denominator) } \\
\frac{1}{d_{i}}= & \text { so } \quad d_{i}=
\end{array}
$$

The lens equation can also be used to find the focal length of a lens when the object distance and image distance are both known:

## Example: Finding the Focal Length of a Converging Lens

A 1.0-cm LED is placed 6.0 cm from a converging lens. An image of the LED is formed 8.0 cm from the lens. Find the focal length of the lens.

Givens: $d_{o}=6.0 \mathrm{~cm} \quad d_{i}=8.0 \mathrm{~cm} \quad$ Unknown: $f=$ ?
Select an Equation: $\frac{1}{f}=\frac{1}{d_{i}}+\frac{1}{d_{o}}$
Substitute and Solve: $\frac{1}{f}=\frac{1}{6.0 \mathrm{~cm}}+\frac{1}{8.0 \mathrm{~cm}}$

$$
\begin{aligned}
& \frac{1}{f}=\frac{4}{24 \mathrm{~cm}}+\frac{3}{24 \mathrm{~cm}} \quad \text { (find a common denominator) } \\
& \frac{1}{f}=\frac{7}{24 \mathrm{~cm}} \quad \text { so } \quad f=\frac{24 \mathrm{~cm}}{7}=3.4 \mathrm{~cm}
\end{aligned}
$$

Note that the answer might not be a whole number!
Round to the appropriate number of significant digits.

## Practice: Finding the Focal Length of a Converging Lens

A 2.5-cm light bulb is placed 12.0 cm from a converging lens. An image of the bulb is formed 10.0 cm from the lens. Find the focal length of the lens.
Givens: $\quad d_{o}=$
$d_{i}=$
Unknown: $\quad f=$ ?

Select an Equation: $\frac{1}{f}=\frac{1}{d_{i}}+\frac{1}{d_{o}}$
Substitute and Solve: $\frac{1}{f}=$

$$
\begin{array}{ll}
\frac{1}{f}= & \quad \text { (find a common denominator) } \\
\frac{1}{f}= & \text { so } \quad f=
\end{array}
$$

Note that for a very distant object, $\frac{1}{d_{o}} \approx 0$, so $\frac{1}{f}=\frac{1}{d_{i}}+\frac{1}{d_{o}}$ becomes $\frac{1}{f} \approx \frac{1}{d_{i}}$ so $f \approx d_{i}$.
This is the procedure we will use to determine the focal length of our lens in our lab: we will image a distant object and measure the image distance, which we will assume is approximately the focal length.

The magnification of an object is the ratio of the image height, $h_{i}$, to the object height, $h_{o}$ :

$$
M=\frac{h_{i}}{h_{o}}
$$

This will be equal to the negative ratio of the image distance, $d_{i}$, to the object distance, $d_{0}$ :

$$
M=\frac{h_{i}}{h_{o}}=\frac{-d_{i}}{d_{o}}
$$

## Example: Finding the Height of the Image Formed by a Converging Lens

A $4.0-\mathrm{cm}$ tall light bulb is placed 18 cm from a converging lens having a focal length of 12 cm . Determine the image height.

Givens: $\quad f=12 \mathrm{~cm} \quad h_{i}=4.0 \mathrm{~cm} \quad d_{o}=18 \mathrm{~cm} \quad d_{i}=36 \mathrm{~cm}$ (from previously)
Unknown $\quad h_{i}=$ ?
Select an Equation: $\frac{h_{i}}{h_{o}}=\frac{-d_{i}}{d_{o}}$ becomes $h_{i}=\frac{-d_{i} h_{o}}{d_{o}}$
Substitute and Solve: $\quad h_{i}=\frac{-(36 \mathrm{~cm})(4.0 \mathrm{~cm})}{18 \mathrm{~cm}}$

$$
h_{i}=-12 \mathrm{~cm}
$$

The height is negative because the image is inverted!

## Practice: Finding the Image Formed by a Converging Lens

A $2.0-\mathrm{cm}$ tall candle flame is placed 6.0 cm from a converging lens having a focal length of 4.0 cm . Determine the image height.

Givens: $\quad f=\quad h_{i}=\quad d_{o}=\quad d_{i}=\quad$ (from previously)
Unknown $\quad h_{i}=$ ?
Select an Equation: $\frac{h_{i}}{h_{o}}=\frac{-d_{i}}{d_{o}}$ becomes $h_{i}=\frac{-d_{i} h_{o}}{d_{o}}$

Substitute and Solve: $h_{i}=$

$$
h_{i}=
$$

Both the lens equation and the magnification equation work with diverging lenses too. With diverging lenses, the focal length is negative and the image distance will be negative too since the image turns up on the same side as the object instead of on the other side of the lens.

## Example: Finding the Image Formed by a Diverging Lens

A $4.0-\mathrm{cm}$ tall light bulb is placed 12 cm from a diverging lens having a focal length of -6.0 cm . Determine the image distance.

Givens: $\quad f=-6.0 \mathrm{~cm} \quad d_{o}=12 \mathrm{~cm} \quad$ Unknown $d_{i}=$ ?
Select an Equation: $\frac{1}{f}=\frac{1}{d_{i}}+\frac{1}{d_{o}}$ becomes $\frac{1}{d_{i}}=\frac{1}{f}-\frac{1}{d_{o}}$
Substitute and Solve: $\quad \frac{1}{d_{i}}=\frac{1}{-6.0 \mathrm{~cm}}-\frac{1}{12 \mathrm{~cm}}$

$$
\begin{aligned}
& \frac{1}{d_{i}}=\frac{-2}{12 \mathrm{~cm}}-\frac{1}{12 \mathrm{~cm}} \quad \quad \text { (find a common denominator) } \\
& \frac{1}{d_{i}}=\frac{-3}{12 \mathrm{~cm}} \quad \text { so } \quad d_{i}=\frac{12 \mathrm{~cm}}{-3}=-4.0 \mathrm{~cm}
\end{aligned}
$$

## Practice: Finding the Image Formed by a Diverging Lens

A 2.0-cm tall candle flame is placed 6.0 cm from a diverging lens having a focal length of -4.0 cm . Determine the image distance.

Givens: $f=\quad d_{o}=\quad$ Unknown $d_{i}=$ ?
Select an Equation: $\frac{1}{f}=\frac{1}{d_{i}}+\frac{1}{d_{o}}$ becomes $\frac{1}{d_{i}}=\frac{1}{f}-\frac{1}{d_{o}}$
Substitute and Solve: $\frac{1}{d_{i}}=$

$$
\begin{array}{ll}
\frac{1}{d_{i}}= & \quad \text { (find a common denominator) } \\
\frac{1}{d_{i}}= & \text { so } \quad d_{i}=
\end{array}
$$

What would the image height be?

