Elastic Potential Energy Homework Solutions

p. 207 #5 $m=1.37\,kg$ $k=5.20\times10^{2}\frac{N}{m}$

(a) $x=?$

$$\Sigma F=0$$

$$k\,x-m\,g=0$$

$$x=\frac{m\,g}{k}=\frac{(1.37\,kg)(9.80\frac{m}{s^2})}{5.20\times10^{2}\frac{N}{m}}=0.0258\,m$$

(b) $x=0.0159\,m$ $\Sigma F=?$

$$\Sigma F=k\,x-m\,g=(5.20\times10^{2}\frac{N}{m})(0.0159\,m)-(1.37\,kg)(9.80\frac{m}{s^2})=-5.16\,N$$

i.e. 5.16 N [down]

(c) $x=0.0205\,m$ $a=?$

$$a=\frac{\Sigma F}{m}=\frac{k\,x-m\,g}{m}=\frac{(5.20\times10^{2}\frac{N}{m})(0.0205\,m)-(1.37\,kg)(9.80\frac{m}{s^2})}{1.37\,kg}=-2.02\frac{m}{s^2}$$

i.e. 2.02 m/s^2 [down]

p. 211 #10 $m=0.0078\,kg$ $k=3.5\times10^{2}\frac{N}{m}$ $x=0.045\,m$

(a) $E_e=?$

$$E_e=\frac{1}{2}k\,x^2=\frac{1}{2}(3.5\times10^{2}\frac{N}{m})(0.045\,m)^2=0.35\,J$$

(b) $v_2=?$

$$E_{T1}=E_{T2}$$

$$\frac{1}{2}k\,x_1^2=\frac{1}{2}m\,v_2^2$$ (since E_{e1} and E_{k2} are the only non-zero energies)

$$v=\sqrt{\frac{k\,x^2}{m}}=\sqrt{\frac{(3.5\times10^{2}\frac{N}{m})(0.045\,m)^2}{0.0078\,kg}}=9.5\frac{m}{s}$$

p. 211 #12 $m = 0.20\,kg$ $k = 55\,\frac{N}{m}$

(a) $\Delta h = x_2 = 0.015\,m$ $v_2 = ?$

Some of the gravitational potential energy has been converted to kinetic and elastic:

$$\Delta E_g = E_{k2} + E_{e2}$$

$$m\,g\,\Delta h = \frac{1}{2}m\,v_2^2 + \frac{1}{2}k\,x_2^2$$

Solving for v_2 ,

$$v_2 = \sqrt{\frac{2\,m\,g\,\Delta h - k\,x_2^2}{m}} = \sqrt{\frac{2(0.20\,kg)(9.8\,\frac{m}{s^2})(0.015\,m) - (55\,\frac{N}{m})(0.015\,m)^2}{0.20\,kg}} = 0.48\,\frac{m}{s}$$

(b) $x_{max} = ?$

At maximum extension, speed is zero and there is no kinetic energy:

$$\Delta E_g = E_{emax}$$

$$m\,g\,\Delta h = \frac{1}{2}x_{max}^2$$

$$m\,g\,x_{max} = \frac{1}{2}k\,x_{max}^2$$

$$\frac{1}{2}k\,x_{max}^2 - m\,g\,x_{max} = 0$$

$$\frac{1}{2}(55\,\frac{N}{m})\,x_{max}^2 - (0.20\,kg)(9.8\,\frac{m}{s^2})\,x_{max} = 0$$

The non-zero solution is $x_{max} = 0.071\,m$

(The zero solution is simply the moment of release.)

Note that mass matters. This is why it is important to give your correct mass when you go bungee jumping. You want the maximum extension of the bungee to be less than the distance to the ground.

p. 211 #13 $\quad k = 12\,\frac{N}{m} \qquad \Delta d_y = -0.93\,m \qquad m = 0.0083\,kg \qquad x = 0.040\,m$

We want to find the range of the projectile Δd_x .

First, determine the horizontal speed at launch:

$$E_{T1} = E_{T2}$$

$$\frac{1}{2} k x_1^2 = \frac{1}{2} m v_2^2 \qquad \text{(since } E_{e1} \text{ and } E_{k2} \text{ are the only non-zero energies)}$$

$$v = \sqrt{\frac{k x^2}{m}} = \sqrt{\frac{(12\,\frac{N}{m})(0.040\,m)^2}{0.0083\,kg}} = 1.521\,\frac{m}{s}$$

The time for the projectile to fall from its initial height is:

$$\Delta d_y = v_{1y} \Delta t - \frac{1}{2} g \Delta t^2 \;\rightarrow\; \Delta d = \sqrt{\frac{-2\,\Delta d_y}{g}} = \sqrt{\frac{-2(-0.93\,m)}{9.8\,\frac{m}{s^2}}} = 0.4357\,s$$

The range of the projectile is:

$$\Delta d_x = v_x \Delta t = (1.521\,\frac{m}{s})(0.4357\,s) = 0.66\,m$$